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Numerical simulation of laminar flow of water-based magneto-rheological fluids in microtubes with wall roughness effect☆

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Abstract

Fully developed laminar flows of water-based magneto-rheological (MR) fluids in microtubes at various Reynolds and Hedsrom numbers have been numerically simulated using finite difference method. The Bingham plastic constitutive model has been used to represent the flow behavior of MR fluids. The combined effects of wall roughness and shear yield stress on the flow characteristics of MR fluids, which are considered to be homogeneous by assuming the small particles with low concentration in the water, through microtubes have been numerically investigated. The effect of wall roughness on the flow behavior has been taken into account by incorporating a roughness–viscosity model based on the variation of the MR fluid apparent viscosity across the tube. Significant departures from the conventional laminar flow theory have been acquired for the microtube flows considered.

Keywords: Microchannel; Microtube; MR fluid; ER fluid; Bingham plastic; Wall roughness; Laminar flow

1. Introduction

Magneto-rheological (MR) fluids are dispersions of fine (~0.05–10 μm) magnetically soft, multi-domain particles [1]. The field-induced transition of these smart fluids from the liquid to a geleous state

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is fast and reversible, i.e. after switching on the field the stiffening of the fluid occurs within a couple of milliseconds and after removal of the field, the material returns to its original fluid state. The degree of the stiffness depends on the kind of material and on the strength of the electric or magnetic field, respectively, and can therefore be regulated by the field. Correspondingly, electro-rheological (ER) fluids are colloidal suspensions, which exhibit dramatic reversible changes in properties when acted upon by an electric field. Both MR and ER fluids develop significant increases in shear yield stress and viscosity when they subjected to the electric field. They also behave like Newtonian fluids under zero field condition. With the field applied, these fluids become like non-Newtonian fluids for which the shearing stress can be represented by either Bingham plastic or Herschel–Bulkley constitutive model. This means that these fluids exhibit a finite yield stress with the shear stress depending upon the shear rate. These outstanding properties of such smart fluids give them a large potential for a variety of technical applications from which interesting perspectives.

MR fluid devices are being used and developed for shock absorbers, clutches, brakes, actuators, exercise equipment, and seismic dampers [2–6]. For such fluids, water, some hydrocarbons, glycol and silicone oil are generally employed as the carrier liquid depending upon the requirements of the application considered. Interest in such controllable fluids derives from their ability to provide simple, quiet, rapid-response interfaces between electronic controls and mechanical systems. These controllable fluids have the potential to radically change the way electromechanical devices that are designed and operated have long been recognized [7]. In the future, they also may play an important role in the "chemistry laboratory on a chip" systems currently under development [8]. However, before such microfluidic applications can be designed, researchers need more information about how MR fluids behave at the microscopic level.

A detailed literature review would reveal that the understanding of flow behavior of both Newtonian and particularly non-Newtonian fluids through microchannels is far from complete and inconclusive. The present work is a preliminary study of the flow behavior of water-based MR fluid through microtubes using Bingham plastic constitutive model. This study focuses on the effects of the wall roughness and yield stress on the flow behavior. The analytical solution of the laminar Bingham plastic fluid flow is first introduced, and a roughness–viscosity model proposed by Mala [9] is adapted to account for variation of apparent viscosity of the MR fluid across a microtube. The governing differential equation describing the flow in the microtubes is solved numerically using finite difference method (FDM).

2. Flow analysis of MR fluid in a round tube using Bingham plastic model

For one-dimensional steady flow in a circular tube, the streamlines are parallel to the wall, so that velocity can be assumed to vary in the radial direction only, i.e., $u = u(r)$. For this case, z-component of the momentum equation with the gravity neglected in cylindrical coordinates for steady-state conditions reduces to:

$$\tau_{\mathrm{rz}} - \frac{1}{2} r \frac{\mathrm{d}p}{\mathrm{d}z} \tag{1}$$

The Bingham plastic constitutive model has shown to be applicable to represent the flow behavior of rheological materials such as MR and ER fluids. According to this model, the flow is generally divided into pre-yield and post-yield regions, depending on whether the material is stressed below or above a

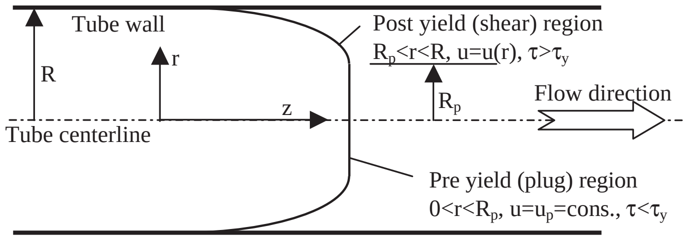

Fig. 1. Typical velocity profile in a circular tube for a Bingham plastic flow.

yield stress value (τ_y). This is shown in Fig. 1 schematically. In the pre-yield region, a plug flow occurs with a plug radius, where the velocity of the fluid remains at a constant value as show in Fig. 1. Plug radius R_p can be determined by:

$$R_p = \frac{2\tau_y}{|dp/dz|} \tag{2}$$

When the shear stress exceeds the yield stress, the material is in the post-yield region. This implies that the shear stress in the material must exceed the dynamic yield stress before it can flow. For the Bingham plastic model, the total shear stress (τ_{rz}) is given by

$$\left.\begin{array}{ll} \tau_{rz} = \tau_y \mathrm{Sgn}\left(\frac{du}{dr}\right) + \mu_0 \frac{du}{dr}, & |\tau_{rz}|>|\tau_y| \\ \frac{du}{dr} = 0, & |\tau_{rz}|<|\tau_y| \end{array}\right\} \tag{3}$$

where μ_0 is the plastic viscosity, and is assumed to be constant. As can be seen from Fig. 1, the post-yield flow region occurs for $R_p \le r \le R$. Considering $du/dr < 0$ in this region, and combining Eqs. (1) and (3), one would get:

$$-\tau_y + \mu_0 \frac{du}{dr} = -\frac{1}{2} r \frac{dp}{dz} \tag{4}$$

Solving Eq. (4) with $u=0$, at $r=R$ (no-slip on the wall), one can obtain the velocity distribution in the post-yield region as follows:

$$u(r) = \frac{1}{4\mu_0}\frac{dp}{dz} R^2 \left(\frac{r^2}{R^2} - 1\right) + \frac{\tau_y R}{\mu_0}\left(\frac{r}{R} - 1\right), \quad (R_p \le r \le R) \tag{5}$$

The volume flow rate through a circular tube, for the Bingham plastic fluid with fully developed, steady flow and no-slip condition assumptions is expressed as:

$$Q = \pi R_p^2 u_p + \int_{R_p}^{R} u(r) dA \tag{6}$$

Fanning friction factor that is widely used to describe flow friction is given by:

$$f = \frac{\Delta p}{2\rho U_m^2} \frac{D}{L} \tag{7}$$

where $U_m = (Q/\pi R^2)$ is the average velocity.

3. Roughness–viscosity model for MR fluid flow in microtubes and solution procedure

The effect of tube wall roughness on the laminar flow in macro-scale circular tubes has been ignored and for a Newtonian fluid the friction factor is assumed to be a function of only the *Re* number. However, the presence of surface roughness affects the laminar velocity profile when the fluid is flowing through micro-scale tubes or channels. This phenomenon has been illustrated by a number of experiments and a comprehensive review can be found in the literature [10–12]. In order to consider the effects of surface roughness on laminar flow in microtubes, Mala [9] proposed a roughness–viscosity function based on Merkle's [10] modified viscosity model as follows:

$$\frac{\mu_R}{\mu_0} = ARe_\varepsilon \frac{r}{\varepsilon}\left[1-\exp\left(-\frac{Re_\varepsilon}{Re}\frac{r}{\varepsilon}\right)\right]^2 \tag{8}$$

where μ_R is the roughness viscosity, ε is the wall roughness and the roughness Reynolds number (Re_ε) is defined by $Re_\varepsilon=(\varepsilon^2/\nu)(\mathrm{d}u/\mathrm{d}r)_{r=R}$, R is the tube radius and A is a coefficient, which depends on the relative roughness height (ε/R). In this study, the roughness–viscosity relation is combined with Eq. (4) to account for the effect of roughness on the laminar flow of fluid with yield stress in microtubes. Some assumptions must be made before proceed to develop the model since an MR fluid flow is actually a two-phase flow. These are (i) the diameter of the particles in the MR fluid are fine enough compared to tube size and roughness height, (ii) the concentration of the particles are low enough to ensure a homogenous fluid content, (iii) microtube is thought to be vertical to avoid particles from settling toward the tube wall, and (iv) wall roughness effect are independent upon MR effects across the microtube.

By adding the roughness viscosity in the momentum equation in a manner similar to the eddy viscosity in turbulent flow, and introducing non-dimensional terms, Eq. (4) becomes:

$$\left[1 + ARe\bar{\varepsilon}\bar{r}\left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{r}}\right|_{\bar{r}=1}\frac{\bar{r}}{\bar{\varepsilon}}\left(1-\exp\left(-\bar{\varepsilon}\bar{r}\left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{r}}\right|_{\bar{r}=1}\right)\right)^2\right]\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{r}} - \frac{Re}{8}\frac{\mathrm{d}\bar{p}}{\mathrm{d}\bar{p}}\bar{r} - \frac{He}{2Re} = 0 \tag{9}$$

where: $Re=(\rho U_m D/\mu_0)$; $He=(D^2\rho\tau_y/\mu_0^2)$; $\bar{u}=u/U_m$; $\bar{r}=(r/D/2)$; $\bar{\varepsilon}=(\varepsilon/D)$; $\bar{p}=(p/1/2\rho U_m^2)$.

As can be seen from the selected dimensionless parameters, *Re* represents the Newtonian behavior of the fluid and tube geometry, *He* represents the effect of yield stress. Eq. (9) is a modified momentum equation that includes the effect of tube wall roughness in a laminar flow of MR fluids exhibiting Bingham plastic behavior. It is a first order non-linear differential equation, which does not have a closed-form solution, and can be solved using finite difference method (FDM). The dimensionless velocity gradient in Eq. (9) is written in backward difference form for $R_p \le r \le R$, and the resulting non-linear system of equations is solved by using the Newton–Raphson method.

4. Results and discussion

First, the accuracy of the numerical analysis is verified by solving the Eq. (9) for the case of $He=Re_\varepsilon=0$ to obtain the velocity distribution. The results are compared to the analytical solution given in the literature for the laminar, fully developed flow of a Newtonian fluid in a circular tube. The number of nodes is selected such that to

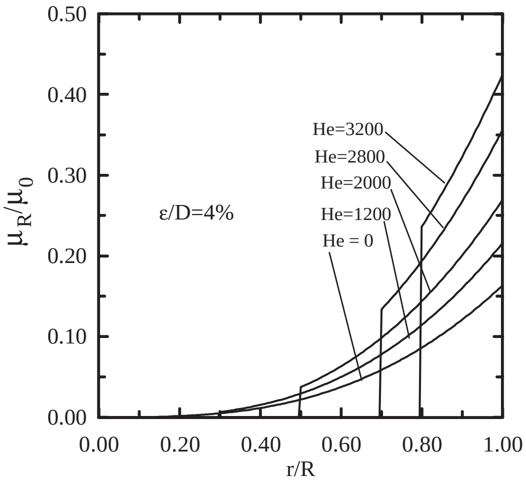

Fig. 2. Variation of roughness viscosity with local radius for different *He* numbers.

ensure an average error of less than 0.01%. A no-slip boundary condition is assumed on the tube wall, and a symmetric flow is considered about the centerline of the tube.

It is generally accepted that the surface roughness has an effect on laminar flow characteristics and results in a reduction in *Re* number [10,11]. Based on the Merkle's [10] modified viscosity model, the roughness-affected viscosity (μ_R) increases exponentially from the tube centerline to the wall. At the centerline, the roughness viscosity is assumed to be zero, and at the wall it reaches its maximum value. The average height of the surface roughness dictates the dominance of this additional frictional effect.

Fig. 2 shows the effect of *He* number (or yield stresses) on the roughness viscosity for a constant roughness height. It can be concluded that the roughness viscosity increases non-linearly with increasing *He* number. For instance, roughness dependent viscosity is increased by 169% with the increase in *He* number

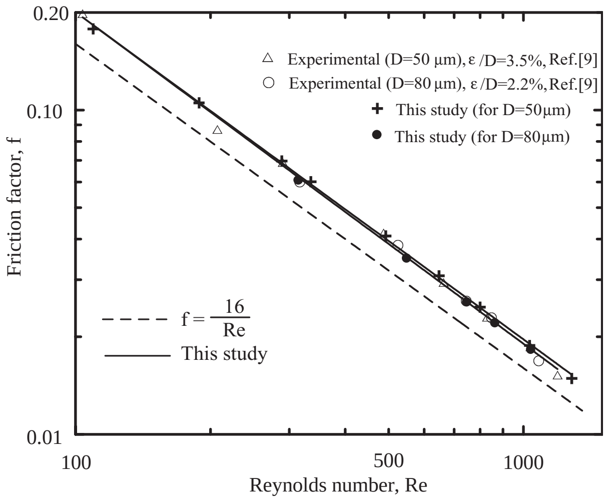

Fig. 3. Comparison among theoretical, experimental and model results.

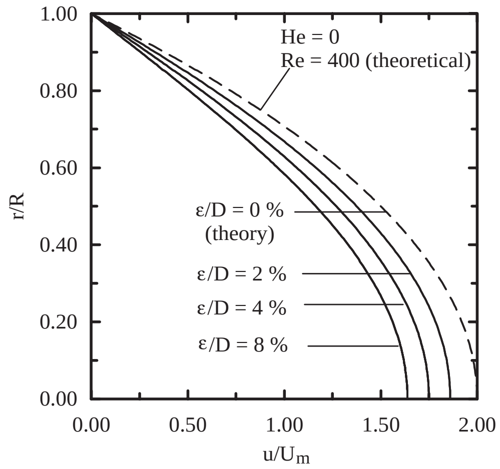

Fig. 4. Effect of relative roughness on velocity profile for $He = 0$.

from 0 to 3200. A non-zero *He* number introduces the plug flow and increasing *He* results in increased radius of the plug region. This results in increased velocity gradient. Therefore, near the wall, the local interactions between fluid particles and surface geometry become more significant, which may cause additional frictional effects.

Comparison between the experimentally measured and predicted friction factors is shown in Fig. 3, from which can be seen that the effect of wall roughness plays an important role on the friction factor even in the laminar flow conditions. The dashed line indicates theoretical friction factor for Newtonian fluid, which is known as $f \cdot Re = 16$. From Fig. 3, it can be concluded that there is a good agreement between the predictions of the present study and the experimental friction factor data taken from the literature [9].

Fig. 4 illustrates non-dimensional velocity profiles, as a function of relative surface roughness for $He = 0$. Velocity values are non-dimensionalized using the average velocity for the smooth pipe for a given pressure gradient. The dashed parabolic profile indicates the theoretical velocity profile of a laminar flow in a circular tube.

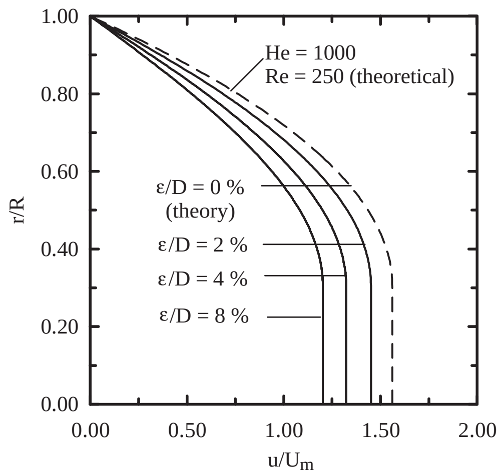

Fig. 5. Effect of relative roughness on velocity profile for $He = 1000$.

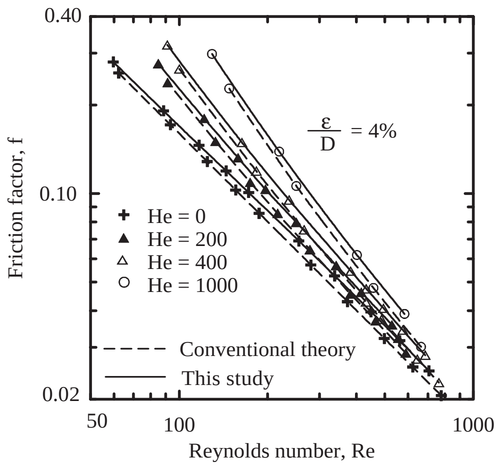

Fig. 6. Variation of friction factor ratio with *Re* number for different *He* numbers ($\varepsilon/D = 4\%$).

The other three profiles are generated from the model presented in this study. It is clear from Fig. 4 that the maximum and average velocities predicted by the model used in this study reduce as the relative roughness increases. For example, the centerline velocity reduces by 6.5%, 12.5% and 18.7% compared to the conventional laminar flow theory for roughness ratios of 2%, 4% and 8%, respectively. Therefore flow rate, *Re* number and the velocity gradient near the wall decreases with increasing relative roughness.

A similar trend is seen in Fig. 5, which shows the velocity profiles for different relative surface roughness values and a constant *He* number of 1000. The dashed curve represents the theoretical laminar Bingham plastic flow velocity profile, which is obtained from Eq. (5). As in the case of $He = 0$, the peak velocity (or plug velocity) decreases considerably with increasing roughness. For the cases considered in Fig. 5, peak velocity reduces by 7.2%, 15.5% and 23.2% compared to the conventional theory for relative roughness of 2%, 4% and 8%, respectively. If the peak velocity reductions for zero (Fig. 4) and non-zero (Fig. 5) *He* numbers are compared for

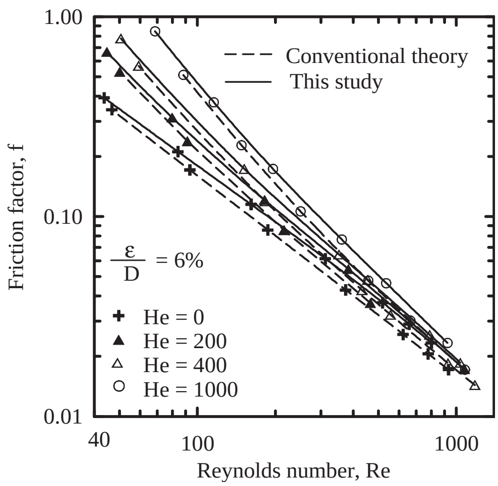

Fig. 7. Variation of friction factor ratio with *Re* number for different *He* numbers ($\varepsilon/D = 6\%$).

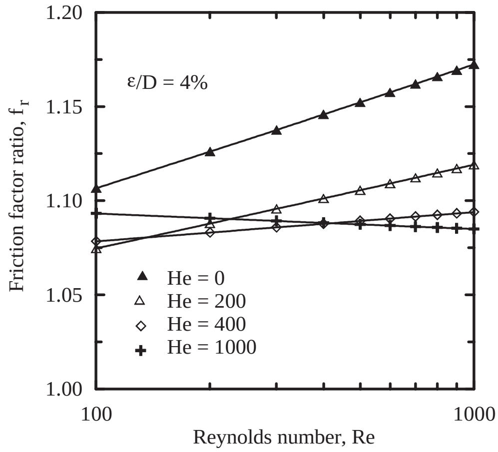

Fig. 8. Effect of *He* number for constant relative roughness of 4%.

the same surface roughness values, one can conclude that the existence of yield stress increases the peak velocity reduction for the same relative roughness. For example, the maximum velocity reductions for 8% roughness are 18.7% and 23.2% for $He=0$ and $He=1000$, respectively.

Fig. 6 illustrates the variation of friction coefficient with *Re* number as a function of *He*. The results show that the difference between friction factors for different *He* decreases with increasing *Re*. A similar trend is also seen in Fig. 7, for which the relative roughness is 6%. On the other hand, deviation of friction factor for a rough tube from a smooth one increases with increasing *Re* number with the exception of $He=1000$. This is probably due to the increase in the interactions between fluid particles and irregular wall surface as discussed before. The deviation also increases with increasing in the roughness as expected.

Another important parameter to evaluate the relative effect of wall roughness can be defined as:

$$f_r = \frac{(fRe)_{\varepsilon \neq 0}}{(fRe)_{\varepsilon = 0}} \tag{10}$$

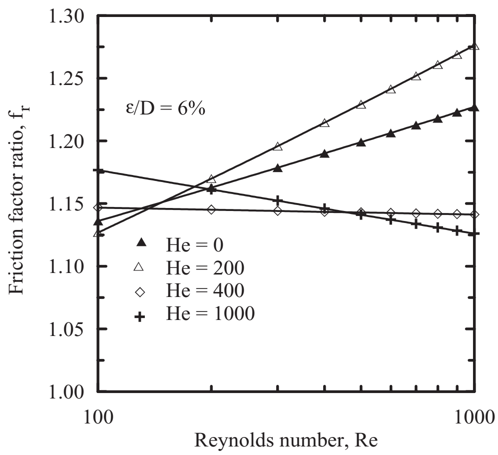

Fig. 9. Effect of *He* number for constant relative roughness of 6%.

Considering the same Re numbers, Eq. (10) becomes:

$$f_r = \frac{f_{\varepsilon \neq 0}}{f_{\varepsilon = 0}} \tag{11}$$

In conventional laminar-flow theory, based on macro tubes, the friction factor ratio, given by Eq. (11), is $f_r = 1$. The friction factor ratio, f_r, as a function of Re number for two different roughness ratios, are presented in Figs. 8 and 9. These results indicate that f_r is greater than unity and it varies with Re number for microtubes. Similar results also have been reported in the literature [9–11]. Also, the deviation from the conventional theory increases as the He number decreases, for $Re > \sim 400$. Thus, it can be deduced that the deviation from the conventional theory increases with an increase in the Re number, for low He numbers, such as 200. After exceeding a certain He number, it decreases with the increase in Re number.

5. Conclusions

Fully developed laminar water-based MR fluid flows in microtubes at various Reynolds and Hedsrom numbers and relative roughness heights have been numerically simulated by using finite difference method. A modified roughness–viscosity model has been adopted to account for the effect of wall roughness, which assumes a radial variation in the apparent viscosity across the microtube. Non-Newtonian behavior of the MR fluid has been represented by Bingham plastic model. The numerical results showed that significant depressions occurred in the conventional laminar flow velocity profile as the relative roughness heights increased. The departure from the conventional laminar flow theory has been shown to be dependent upon MR fluid yield stress, which is characterized by Hedsrom number. The friction factor ratios have been obtained remarkably larger than unity depending on the roughness height Reynolds, and Hedsrom numbers. Therefore, the combined effects of wall roughness and the yield stress exhibited to have a considerable impact on the flow behavior of water-based MR fluids through microtubes.

Nomenclature

A	A coefficient
D	Tube diameter, m
dp/dz	Pressure gradient, Pa/m
f	Fanning friction factor
f_r	Friction factor ratio
He	Hedsrom number
L	Tube length, m
Q	Volume flow rate, m^3/s
r	Radial coordinate, m
R	Tube radius, m
Re	Reynolds number
Re_ε	Reynolds number at the wall
R_p	Plug radius, m
u	Velocity, m/s

U_m Average velocity, m/s
u_p Plug velocity, m/s
ρ Density of water, kg/m^3
v Kinematic viscosity of water, m^2/s
μ_0 Dynamic viscosity of water, Pa·s
Δp Pressure drop, Pa
μ_R Dynamic viscosity of water near wall, Pa·s
τ_{rz} Shear stress, Pa
τ_y Yield stress, Pa
ε Roughness height, m

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